

Midterm 2 Samples

Sample 1

1. Find the prime factorization of $11!$.
2. Let $a_1 = \frac{1}{2}$ and $a_n = a_{n-1} + \frac{1}{n(n+1)}$ for $n \geq 2$. Conjecture a formula for a_n and use induction to prove the conjectured formula.
3. Find the solutions of the congruence $15x^2 + 19 \equiv 5 \pmod{11}$.
4. Which integers leave a remainder of 1 when divided by 2 as well as when divided by 3 or 5? Show your work.
5. Explain why the function $f(x) = (13x + 1) \pmod{26}$ would not be a good coding function.
6. How many different license plates can be made if each license plate consists of three letters followed by three digits or four letters followed by two digits?
7. Find the (multiplicative) inverse of 12 module 45 or show why it doesn't have an inverse.
8. Prove or disprove: if $a \equiv b \pmod{m^2}$ then $a \equiv b \pmod{m}$.
9. Show that the function $f(x) = (x + 2) \log(x^2 + 1) + \log(x^3 + 1)$ is $O(x \log x)$.

Sample 2

1. Let $a_1 = 2$, $a_2 = 9$, and $a_n = 2a_{n-1} + 3a_{n-2}$ for $n \geq 3$. Show that $a_n \leq 3^n$ for all positive integers n .
2. Prove that $2^n > n^2$ for every integer greater than 4.
3. Find $(25^{1202} + 3)^2 \pmod{61}$.
4. Decrypt the received integer 20 if the original message was encrypted using RSA with public keys $(n, e) = (65, 5)$.
5. Solve the linear congruence $54x \equiv 12 \pmod{73}$.
6. Find the (multiplicative) inverse of 15 module 46 or show why it doesn't have an inverse.
7. Either find an integer x such that $x \equiv 2 \pmod{6}$ and $x \equiv 3 \pmod{9}$ are both true, or else prove that there is no such integer.
8. Prove or disprove: there are six consecutive composite integers.
9.
 - (a) How many functions are there from a set with 3 elements to a set with 8 elements?
 - (b) How many 1-1 functions are there from a set with 3 elements to a set with 8 elements?
 - (c) How many onto functions are there from a set with 3 elements to a set with 8 elements?
10. Show that $\sum_{k=1}^n (k^3 + k)$ is $\Theta(n^4)$.